



MARITIME  
APPRENTICESHIP  
PROGRAM

# Basic Mathematics Workbook





## Working with Basic Mathematics

### *Introduction*

Thank you for applying to join the Workboat Academy. This new and exciting program will develop your skills, knowledge and understanding so that you can be successful as a professional mariner.

Basic math skills are an important element of everyday shipboard life. You will need a solid foundation of these skills in order to succeed in this training program. Therefore, as a prerequisite to being admitted into our program, you have to complete the “Math Competency Exam”. This Workbook will help you prepare for that exam.

The purpose of the exam is to enable you to self-screen your skills and abilities against those that are reflected and expected in the coursework of the Workboat Academy. It is imperative that you appreciate the importance of the exam and recognize the level of math aptitude that is necessary to successfully complete your intended program.

Please note that the distribution of the exam is limited; do not copy or further disseminate the exam.

“Working With Mathematics” is a self study guide designed to teach or refresh basic math skills needed for the Workboat Academy. It is divided into eight training modules. Each training module includes example problems with solutions, as well as additional online references to further assist you with the skills presented in that specific module. It is important that you understand each training module and how to work the various problems before you take the Math Competency Exam.

In addition to this Workbook and the suggested websites, it is highly recommend that you seek additional reference from the following text: Formulae for the Mariner by Richard Plant. Formulae for the Mariner is an inexpensive resource that may aid in the fine tuning of your skills for this Math Exam, and it will also serve as a valuable reference for your future career as a mariner.



## Modules

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**Note:**

*It is highly recommended that you purchase your own calculator. The National Maritime Center now only allows the use of Texas Instruments TI-30XIIS calculator. Being familiar with your own personal calculator will greatly assist your success.*



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## 1. Order of Operations

The first step to solving equations and applying formulas is the basic step of understanding the correct order of operations. Knowing and using the correct sequence of steps is the best way to ensure you are on the path to correctly solving mathematical problems.

The order of operations is as follows:

1. Solve all the operations that lie inside brackets and parentheses
2. Solve all operations involving exponents or radicals
3. From left to right, solve all multiplication and division
4. From left to right, solve all addition and subtraction

When a problem contains multiple operations, it is imperative that you follow the order of operations because solving the operations in different sequences will produce different results.

*Example #1:* Solve the following, using the correct order of operations.

$$7 - 8 + 2 = X$$

Solving from left to right, we can see that:

$$\begin{aligned}7 - 8 + 2 &= X \\ -1 + 2 &= X \\ 1 &= X\end{aligned}$$

*Example #2:* Solve the following, using the correct order of operations. This time note the use of parenthesis.

$$7 - (8 + 2) = X$$

Solve the operations inside the parenthesis first.

$$\begin{aligned}7 - (8 + 2) &= X \\ 7 - (10) &= X\end{aligned}$$

Then solve the remaining operation.

$$\begin{aligned}7 - (10) &= X \\ -3 &= X\end{aligned}$$

As you can see in the Examples, changing the order of operations and/or adding parenthesis to an equation changes the result. In order to obtain the correct result, you must remember and perform the correct order of operations.



For additional reference, please visit:

[www.khanacademy.org](http://www.khanacademy.org)

- Arithmetic & Pre-algebra
- Addition & Subtraction of numbers
- Multiplication & Division of numbers
- Negative numbers & absolute value
- Negative numbers basics
- Adding and subtracting negative numbers
- Multiplying & dividing negative numbers



***Practice Problems #1:*****Solve the following:**

1.  $-2 + 9 = \underline{\quad}$
2.  $-2 - 9 = \underline{\quad}$
3.  $-2 - (-9) = \underline{\quad}$
4.  $4 - 5(6 - 1) = \underline{\quad}$
5.  $(5 - 3)^2 + 4 = \underline{\quad}$
6.  $[8(6 - 2)] + 3 = \underline{\quad}$
7.  $(81 \div 9) - 2 = \underline{\quad}$
8.  $5 + (-10) - 3 = \underline{\quad}$
9.  $7 + 4(9-5) + 3(8-5) = \underline{\quad}$
10.  $9 - 2 \times 6 = \underline{\quad}$





**Answers to Practice Problems #1**

1. 7
2. -11
3. 7
4. -21
5. 8
6. 35
7. 7
8. -8
9. 32
10. -3



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## 2. Solving Equations

After mastering the order of operations, you can solve basic algebraic equations where there is an unknown value called a variable. In this Workbook, the variable will be  $X$ ; thus, we are solving equations for  $X$ . However, variables may be represented by any letter of the alphabet. In a single equation, the value of the variable must remain the same throughout the problem.

In order to solve for  $X$ , you want to isolate  $X$  on one side of the equation. It is helpful to think of the equation as something you want to keep balanced. If you perform one operation on one side of the equation (in an effort to isolate  $X$ ), then you must perform that same operation on the other side of the equals (=) sign of the equation.

You can always check your solution by plugging in your answer in the equation where  $X$  appears. Then, solve the equation and ensure the equation (and thus your answer for the variable) is correct.

*Example #1:* Solve the following for  $X$ .

$$X^2 - 49 = 0$$

To isolate  $X$ , we can add 49 to both sides of the equation.

$$X^2 = 49$$

To further isolate  $X$ , we can take the square root of both sides of the equation.

$$X = \sqrt{49}$$

$$X = \pm 7$$

Because  $(+7)^2$  and  $(-7)^2$  both equal 49, in this problem,  $X = +7$  and  $-7$

*Example #2:* Solve the following for  $X$ .

$$X + 5 = 2X$$

To isolate  $X$ , we can subtract  $X$  from both sides of the equation.

$$5 = 2X - X$$



We can now simplify (by subtraction) on the right side:

$$5 = X$$

For additional reference, please visit:

[www.khanacademy.org](http://www.khanacademy.org)

- Algebra
- Introduction to Algebra
- Linear equations
- Systems of equations inequalities
- Simple equations



***Practice Problems #2:***

1.  $X + 23 = 2X$

2.  $X^2 - 81 = 0$

3.  $X \div 3 = 20$

4.  $3X + 8 = -2X + 9$

5.  $4X = 16$



**Answers to Practice Problems #2**

1. 23
2.  $\pm 9$
3. 60
4.  $1/5$
5. 4



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### 3. Working with Basic Formulas

Working with and understanding basic formulas and how to arrange them is an important skill necessary in today's world, especially for an officer on a modern commercial vessel.

Let's look at a basic formula to calculate Distance, Time, and Speed. The simple formula is:  $D = S \times T$ .

Where: D = Distance (usually expressed in nautical miles)

S = Speed (usually expressed in knots)

T = Time (expressed in hours and/or minutes, or tenths of a minute)

*Example #1:* Your vessel is traveling at 18 knots and you travel for 2 hours. How far did you travel?

Distance = Speed x Time

$D = S \times T$

$D = 18 \text{ kts} \times 2 \text{ hours}$

**$D = 36 \text{ nautical miles (nm)}$**

*Example #2:* Your vessel is traveling at 14 knots and you travel for 3 hours and 15 minutes.

*Note:* Since calculators perform their functions in tenths, and we are working with time, which is in hours, minutes and seconds, we need to convert the 15 minutes to tenths of an hour. This is easy. Simply divide the 15 minutes by 60 (minutes in one hour). The answer is 0.25 of an hour. This makes sense since 15 minutes is one-quarter of one-hour.

The same process is used to convert tenths of a minute. Since there are 60 seconds in a minute, we simply divide the number of seconds by 60 to convert to tenths of a minute. 54 seconds is how many tenths of a minute?

$54 \div 60 = 0.9$  tenths. To check your results multiply  $0.9 \times 60 = 54$  seconds.

Seconds / Minutes	Minutes / Hours
6 sec = 1/10 <sup>th</sup> of a minute	6 min = 1/10 <sup>th</sup> of an hour
12 sec = 2/10 <sup>th</sup> of a minute	12 min = 2/10 <sup>th</sup> of an hour
18 sec = 3/10 <sup>th</sup> of a minute	18 min = 3/10 <sup>th</sup> of an hour
24 sec = 4/10 <sup>th</sup> of a minute	24 min = 4/10 <sup>th</sup> of an hour
30 sec = 5/10 <sup>th</sup> of a minute	30 min = 5/10 <sup>th</sup> of an hour
36 sec = 6/10 <sup>th</sup> of a minute	36 min = 6/10 <sup>th</sup> of an hour
42 sec = 7/10 <sup>th</sup> of a minute	42 min = 7/10 <sup>th</sup> of an hour
48 sec = 8/10 <sup>th</sup> of a minute	48 min = 8/10 <sup>th</sup> of an hour
54 sec = 9/10 <sup>th</sup> of a minute	54 min = 9/10 <sup>th</sup> of an hour
60 sec = 10/10 <sup>th</sup> of a minute	60 min = 10/10 <sup>th</sup> or 1 hour





Now back to our problem: Your vessel is traveling at 14 knots and you travel for 3 hours and 15 minutes. How far did you travel?

$$D = S \times T$$

$$D = 14 \text{ kts} \times 3:15$$

$$D = 14 \times 3.25 \text{ (remember to convert minutes to tenths)}$$

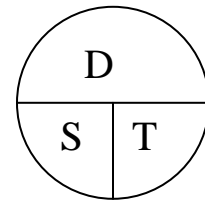
$$\mathbf{D = 45.5 \text{ nm}}$$

How do we find the speed of our vessel? We need to rearrange the basic formula:

$$D = S \times T.$$

In any formula where the answer is the product of two numbers, we can use the following simple diagram to help us rearrange the formula. By rearranging the formula we can solve for any unknown.

Given distance (D) and time (T) we can calculate the speed of travel. In the diagram D is over T, so divide D by T to calculate the speed (S).



The new formula would look like this:

$$S = \frac{D}{T} \text{ or}$$

$S = D \div T$ . The same rules apply to convert minutes to tenths of a minute.

*Example #3:* What speed is your vessel making if you traveled 45 nm in 3:36?

$$S = \frac{D}{T}$$

$$S = \frac{45 \text{ nm}}{3:36 \text{ (36 minutes} \div 60 = 0.6 \text{ of a hour)}}$$

$$S = \frac{45}{3.6}$$

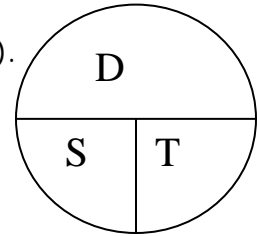
$$\mathbf{S = 12.5 \text{ kts}}$$



How do we find how long it will take for our vessel to travel over a certain distance at a given speed? We need to rearrange the basic formula:  $D = S \times T$ .

In the diagram D is over S, so divide D by S to calculate the speed (S).

The new formula would look like this:



$$T = \frac{D}{S} \text{ or}$$

$$T = D \div S$$

*Example #4:* How long (time) will it take your vessel to reach its destination if you travel 35 nautical miles at a speed of 19 knots?

$$T = \frac{D}{S}$$

$$T = \frac{35}{19}$$

$T = 1.84$  hours. (How many minutes is 0.84 hours?  $0.84 \times 60 = 50.4$  minutes)

**$T = 1$  hour 51 minutes**

We also need to be able to work with positive (+) and negative (-) numbers.

Adding positive numbers:

$$\begin{array}{r} 4.3 \\ (+) \underline{5.8} \\ \hline 10.1 \end{array}$$

Adding negative numbers:

$$\begin{array}{r} -5.4 \\ (+) \underline{-2.7} \\ \hline -8.1 \end{array}$$



For additional reference, please visit:

[www.khanacademy.org](http://www.khanacademy.org)

Science, Physics, Mechanics

-- Solving for time

[www.ehow.com](http://www.ehow.com)

-- Calculating for average velocity or speed

<http://msi.nga.mil>

American Practical Navigator

-- Time, speed and distance,

-- Table 11 explanation, page 559

-- Table 11, page 676



**Practice Problems #3:**

	<b>Speed</b>	<b>Time</b>	<b>Distance</b>
1.	13.6	2 hr 12 min	_____
2.	8.0	1 hr 36 min	_____
3.	6.5	_____	14
4.	_____	4 hr 09 min	44
5.	7.0	3 hr 54 min	_____
6.	10.0	_____	29
7.	_____	9 hr 48 min	107
8.	7.3	8 hr 08 min	_____
9.	9.0	_____	82
10.	_____	10 hr 41 min	57

Convert the following:

- 10.3 min to minutes and seconds \_\_\_\_\_
- 12.23 min to minutes and seconds \_\_\_\_\_
- 15 min 42 seconds to minutes and tenths of a minute \_\_\_\_\_
- 72 minutes to hours and tenths of an hour \_\_\_\_\_
- 2.6 hours to hours and minutes \_\_\_\_\_
- 232 minutes to hours and minutes \_\_\_\_\_

Positive and negative numbers:

$$1. \quad \begin{array}{r} 23.8 \\ + 14.9 \\ \hline \end{array}$$

$$2. \quad \begin{array}{r} -42.8 \\ (+) -17.7 \\ \hline \end{array}$$

$$3. \quad \begin{array}{r} 105 \\ (+) - 28 \\ \hline \end{array}$$



**Answers to Practice Problems #3**

1. 29.9 miles
2. 12.8 miles
3. 2 hr 09 min
4. 10.6 knots
5. 27.3 miles
6. 2 hr 54 min
7. 10.9 knots
8. 59.4 miles
9. 9 hr 07 min
10. 5.3 knots

1. 10 min 18 sec
2. 12 min 14 sec
3. 15.7 min
4. 1.2 hours
5. 2 hrs 36 min
6. 3 hrs 52 min

1. 38.7
2. -60.5
3. 77



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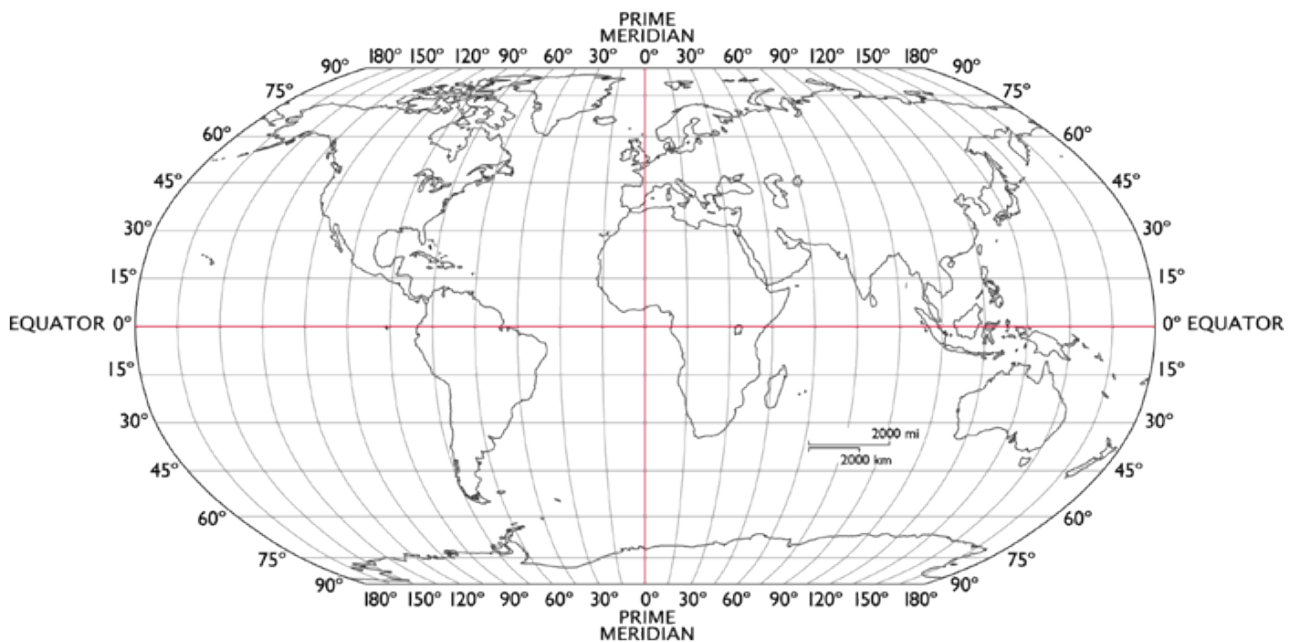
## 4. Latitude and Longitude

Latitude and longitude create a grid system on the globe as one method of identifying location. Latitude and longitude are measured in degrees (°).

Lines of latitude run horizontally and are parallel, because they are equal distance from each other. They are numbered 0 – 90°, and they are labeled as North or South (in relation to the equator, which is 0°).

Lines of longitude run vertically. Although latitude lines can also be called parallels, longitude lines cannot. Instead, they are also known as meridians. They are numbered 0 – 180°, and they are labeled as East or West (in relation to Greenwich, England, which is 0°).

To further identify a specific location on Earth, both longitude and latitude can be measured in degrees, minutes, and seconds. There are 60 minutes in each degree, and each minute is divided into 60 seconds.



Source: [www.about.com](http://www.about.com)

*Example #1:* Your vessel leaves Longitude 15° east and travels due east for 100° of longitude. What is the longitude of arrival?

If you begin at 15° east and are traveling another 100° in the same direction, you add 15° east + 100° east = 115° east. If you look at the map above, you can see



that longitude runs east until  $180^\circ$ . Because our answer is less than  $180^\circ$ , we can see that  $115^\circ$  east is both practical and correct.

*Example #2:* Your vessel leaves Latitude  $15^\circ$  south and travels due north for  $45^\circ$  of latitude. What is the latitude of arrival?

If you begin at  $15^\circ$  south and travel north, you can see that you will be heading toward the Equator. You reach the Equator ( $0^\circ$ ) after traveling  $15^\circ$  north. However, the question says you've traveled north for  $45^\circ$  of latitude. Out of the total of  $45^\circ$  latitude, we have already traveled  $15^\circ$  upon reaching the Equator, so we have another  $30^\circ$  north to travel ( $45^\circ - 15^\circ = 30^\circ$ ). When you travel the remaining  $30^\circ$  north from the Equator ( $0^\circ$ ), you arrive at  $30^\circ$  north.

For additional reference, please visit:

[www.khanacademy.org](http://www.khanacademy.org)

- Math and Trigonometry
  - Radians and degrees
  - Parts of a circle
  - Measuring angles in degrees

[www.wikipedia.org](http://www.wikipedia.org)

- Geographic Coordinate conversion

[www.about.com](http://www.about.com)

- Latitude and Longitude

[www.ehow.com](http://www.ehow.com)

- How to understand Latitude and Longitude
- How to find Latitude and Longitude
- How to read Latitude and Longitude





***Practice Problems #4:***

1. A vessel leaves Longitude  $45^{\circ}$  east and travels due east for  $110^{\circ}$  of longitude.  
What is the longitude of arrival?
2. A vessel leaves Latitude  $35^{\circ}$  north and travels due south for  $50^{\circ}$  of latitude.  
What is the latitude of arrival?
3. A vessel leaves Longitude  $35^{\circ}$  west and travels due east for  $110^{\circ}$  of longitude.  
What is the longitude of arrival?



**Answers to Practice Problems #4**

1.  $155^\circ$  east
2.  $15^\circ$  south
3.  $75^\circ$  east



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## 5. Working with Time

We work with and calculate time in many ways in our everyday life. The ability to accurately add and subtract units of time is an essential skill. Some shipboard operations where this skill is employed include calculating the times of arrivals, optimum tides and currents, and celestial events such as the time of sunrise or sunset. In addition, having a solid grasp of calculating time is crucial to entering many nautical publications.

In the maritime industry we always use the 24-hour clock, or military time:

Midnight = 0000 / 12 Noon = 1200	
1 AM = 0100 / 1PM	= 1300
3 AM = 0300 / 3 PM	= 1500
6 AM = 0600 / 6 PM	= 1800
9 AM = 0900 / 9 PM	= 2100

**Adding Time:** (You may go over 60 minutes, therefore remember to add one hour)

$$\begin{array}{r} 1233 \\ +0621 \\ \hline 1854 \end{array}$$

$$\begin{array}{r} 1742 \\ +0427 \\ \hline 2169 = 2209 \end{array}$$

$$\begin{array}{r} 0656 \text{ (May 1}^{\text{st}}) \\ +1950 \\ \hline 25:106 \text{ min} = 2646 = 0246 \text{ (May 2}^{\text{nd}} \text{ next day)} \end{array}$$

**Subtracting Time:** (You may need to borrow one-hour or 60 minutes)

$$\begin{array}{r} 1233 \\ -0621 \\ \hline 0612 \end{array}$$

$$\begin{array}{r} 1737 \\ -0954 \\ \hline 0743 \end{array} \text{ (borrow 60 minutes from 1700 and add to the 37 min = 97 min. You can then subtract 54 minutes from 97. } 97 - 54 = 43)$$



0239 (May 1<sup>st</sup>)  
- 1951

Solution: In this problem we cannot subtract 1900 from 0200. 0200 is in the morning of the next day, so we need to add 24 hours to the 0200 = 2600. Now we can subtract 19 from 26.

0239 (May 2<sup>nd</sup>) = 2599 (May 2<sup>nd</sup>)  
-1951                      -1951  
**0648 (May 1<sup>st</sup>)**

For additional reference, please visit:

[www.ehow.com](http://www.ehow.com)

- How to find degrees in minutes and seconds
- How to calculate the hours between dates

[www.springfrog.com](http://www.springfrog.com)

- Convert hours, minutes and seconds to decimal time

[www.timeanddate.com](http://www.timeanddate.com)

- Add or subtract hours from a date



***Practice Problems #5:***

Solve the following time problems:

1. 
$$\begin{array}{r} 0344 \text{ May 15th} \\ +0133 \\ \hline \end{array}$$

6. 
$$\begin{array}{r} 1551 \text{ May 15th} \\ -1319 \\ \hline \end{array}$$

2. 
$$\begin{array}{r} 1604 \text{ May 15th} \\ +1414 \\ \hline \end{array}$$

7. 
$$\begin{array}{r} 0438 \text{ May 15th} \\ -0012 \\ \hline \end{array}$$

3. 
$$\begin{array}{r} 1911 \text{ May 15th} \\ +1725 \\ \hline \end{array}$$

8. 
$$\begin{array}{r} 0216 \text{ May 15th} \\ -2323 \\ \hline \end{array}$$

4. 
$$\begin{array}{r} 0150 \text{ May 15th} \\ +2104 \\ \hline \end{array}$$

9. 
$$\begin{array}{r} 1421 \text{ May 15th} \\ -1230 \\ \hline \end{array}$$

5. 
$$\begin{array}{r} 1108 \text{ May 15th} \\ +0837 \\ \hline \end{array}$$

10. 
$$\begin{array}{r} 0912 \text{ May 15th} \\ -1635 \\ \hline \end{array}$$



**Answers to Practice Problems #5**

1. 0517 May 15th
2. 0618 May 16th
3. 1236 May 16th
4. 2254 May 15th
5. 1945 May 15th
6. 0232 May 15th
7. 0426 May 15th
8. 0253 May 14th
9. 0151 May 15th
10. 1637 May 14th



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### 6. Working with Degrees

Doing calculations with degrees is very similar to working with time. Time is expressed in hours, minutes and seconds (or tenths of a minute and seconds). Degrees are expressed in degrees (°), minutes ('), and seconds (").

Degree basics:

60 seconds = 1 minute

60 minutes = 1 degree

10 degrees, 5 minutes, 28 seconds is written as: 10° 05' 28"

As in working with time, we may need to borrow (add or subtract) one degree (or 60 minutes), or one minute (60 seconds). If you have more than 360° as an answer, simply subtract 360°. You may have to add 360° before subtracting from a small number.

#### Adding degrees:

$$\begin{array}{r}
 10^{\circ} 05' 28'' \\
 + 12^{\circ} 52' 13'' \\
 \hline
 22^{\circ} 57' 41''
 \end{array}
 \qquad
 \begin{array}{r}
 28^{\circ} 15' 42'' \\
 + 47^{\circ} 37' 19'' \\
 \hline
 75^{\circ} 53' 01''
 \end{array}
 \qquad
 \begin{array}{r}
 343^{\circ} 27' 19'' \\
 + 31^{\circ} 52' 12'' \\
 \hline
 375^{\circ} 19' 31'' \\
 - 360^{\circ} \\
 \hline
 15^{\circ} 19' 31''
 \end{array}$$

#### Subtracting degrees:

$$\begin{array}{r}
 12^{\circ} 52' 13'' \\
 - 6^{\circ} 48' 09'' \\
 \hline
 6^{\circ} 04' 04''
 \end{array}
 \qquad
 \begin{array}{r}
 47^{\circ} 37' 19'' \\
 - 29^{\circ} 49' 37'' \\
 \hline
 17^{\circ} 47' 42''
 \end{array}
 \qquad
 \begin{array}{r}
 125^{\circ} 16' 27'' = 485^{\circ} (125^{\circ} + 360^{\circ}) \\
 - 237^{\circ} 39' 22'' \\
 \hline
 247^{\circ} 37' 05''
 \end{array}$$

### Working with Circles

Most of us have seen or used a simple handheld compass. A compass circle has 360°. Adding and subtracting these degrees is the same as adding and subtracting other types of degrees. Degrees in a circle are generally expressed in three-digits: 007°, 052°, or 278°. If the addition totals more than 360°, for example 372°, we will need to subtract 360° to get the correct number. 372° - 360° = 012°.

#### Adding degrees:

$$\begin{array}{r}
 258^{\circ} \\
 + 017^{\circ} \\
 \hline
 275^{\circ}
 \end{array}
 \qquad
 \begin{array}{r}
 135^{\circ} \\
 + 278^{\circ} \\
 \hline
 413^{\circ} \\
 - 360^{\circ} \\
 \hline
 053^{\circ}
 \end{array}$$

#### Subtracting degrees:

$$\begin{array}{r}
 225^{\circ} \quad 045^{\circ} = (360^{\circ} + 045^{\circ}) = 405^{\circ} \\
 - 192^{\circ} \quad - 135^{\circ} \\
 \hline
 247^{\circ} \quad 37' \quad 05'' \quad 033^{\circ}
 \end{array}
 \qquad
 \begin{array}{r}
 - 135^{\circ} \\
 \hline
 270^{\circ}
 \end{array}$$



For additional reference, please visit:

[www.ehow.com](http://www.ehow.com)

- How to add degrees, minutes and seconds
- How to calculate degrees minutes and seconds
- How to understand latitude and longitude

[zonalandeduaction.com](http://zonalandeduaction.com)

- Degrees, minutes and seconds



***Practice Problems #6:***

Degrees:

$$\begin{array}{r} 1. \quad 242^\circ 17' 23'' \\ \quad \quad \underline{+17^\circ 42' 18''} \end{array}$$

$$\begin{array}{r} 2. \quad 166^\circ 14' 17'' \\ \quad \quad \underline{-68^\circ 58' 44''} \end{array}$$

$$\begin{array}{r} 3. \quad 352^\circ 01' 49'' \\ \quad \quad \underline{+67^\circ 58' 21''} \end{array}$$

$$\begin{array}{r} 4. \quad 74^\circ 19' 29'' \\ \quad \quad \underline{-5^\circ 42' 17''} \end{array}$$

$$\begin{array}{r} 5. \quad 258^\circ 49' 32'' \\ \quad \quad \underline{-233^\circ 25' 41''} \end{array}$$

Circles:

$$\begin{array}{r} 6. \quad 055^\circ \\ \quad \quad \underline{+29^\circ} \end{array}$$

$$\begin{array}{r} 7. \quad 235^\circ \\ \quad \quad \underline{+136^\circ} \end{array}$$

$$\begin{array}{r} 8. \quad 084^\circ \\ \quad \quad \underline{-97^\circ} \end{array}$$

$$\begin{array}{r} 9. \quad 278^\circ \\ \quad \quad \underline{+147^\circ} \end{array}$$

$$\begin{array}{r} 10. \quad 057^\circ \\ \quad \quad \underline{-149^\circ} \end{array}$$



**Answers to Practice Problems #6**

1.  $259^{\circ} 59' 41''$
2.  $097^{\circ} 15' 33''$
3.  $060^{\circ} 00' 10''$
4.  $068^{\circ} 37' 12''$
5.  $025^{\circ} 23' 51''$
6.  $084^{\circ}$
7.  $011^{\circ}$
8.  $347^{\circ}$
9.  $065^{\circ}$
10.  $268^{\circ}$



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## 7. Working with Tables

Many of the tables used for navigation contain numbers that increase or decrease steadily. Unfortunately, sometimes the number we need does not correspond exactly to the number in the tables, but lies between two numbers or values. Therefore, we need to know how to do basic interpolation in order to derive the exact number required.

Below is a table. In the left column (A) is a series of numbers that are 30° apart. In the right column (B) are values that correspond to those numbers.

A	B
000°	2°
030°	4°
060°	6°
090°	8°

If we were looking for the value for 030° we simply enter the table in column "A" at 030°, go directly across to column "B" and find that the corresponding value is 4°. What if we were seeking the value for 15°? 15° is not in the table, but seeing that 15° is one-half way between 000° and 030°, we can easily guesstimate that the corresponding value for 015° would be 3.0°, or half way between 2 and 4. This is basic visual interpolation.

Problem: What is the corresponding value in column "B" if the desired number in column "A" is 038°?

Solution:

To solve this problem we need to extract three pieces of information:

1. Highlight in the table all of the values surrounding 038° in columns "A" & "B".

A	B
000	2°
030°	4°
060°	6°
090°	8°

2. Find the difference between the numbers in column "A" that surround 038°:

030° base number  
       [038°] desired number

060°

**30°** difference – This is the first piece of information needed.



3. Find the difference in column A between 030° and the number we need, 038°:

030° base number

038° desired number

**8°** difference – This is the second piece of information needed.

4. Find the difference in column B between the values for 030° (4°) and 060° (6°)

4° base number

6°

+**2°** difference (*note: This a positive number since the number is larger than the base number*)

5. Divide the difference found in Step 3 (8°) by the difference in Step 2

(30°)

8°

30° = **0.266°**

6. Multiply 0.266 x +2 (from step three) = 0.53° or **0.5°**. This is the correction that needs to be applied to solve the problem.

$$\frac{8^\circ}{30^\circ} \times 2^\circ = +0.266 \times 2 = +0.53 \text{ or } +\mathbf{0.5^\circ}$$

7. Add this correction number (+0.5°) to the base number in column B (4°)

4.0° base number in column B

+0.5° correction. This number is added since the values in column “B” are increasing

**4.5°**

Answer: **4.5°**

4.5° is the corresponding value in column B for 038° in column A.

A	B
000	2°
030°	4°
<b>038°</b>	<b>4.5°</b>
060°	6°
090°	8°



For additional reference, please visit:

[www.khanacademy.org](http://www.khanacademy.org)

- Algebra
- Linear equations
- Averages
- Proportions
- Ratios





***Practice Problems #7:***

Use the table below to answer the questions

<b>A</b>	<b>B</b>
000°	2°
030°	4°
060°	6°
090°	8°
120°	7°
150°	5°
180°	3°

1. What is the corresponding value in column B for 072° in column A?
2. What is the corresponding value in column B for 167° in column A?



**Answers to Practice Problems #7**

1.  $6.8^\circ$

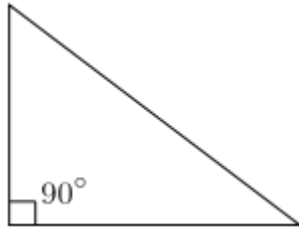
2.  $3.9^\circ$



## 8. Working with Right Triangles

A right triangle, or right angle triangle, is a very common type of triangle. A right triangle is any triangle that has one right angle, or  $90^\circ$ . In navigation, being able to recognize and make calculations based on its properties is very important.

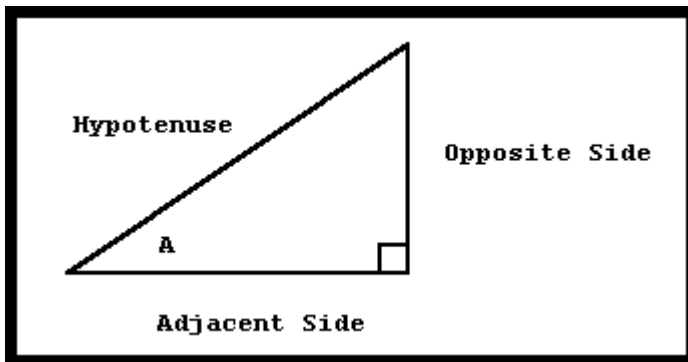
In right triangles, as in all triangles, the sum of all three angles is equal to  $180^\circ$ . Since one of the angles of the right triangle is  $90^\circ$ , the other two angles must be less than  $90^\circ$ . Any angle less than  $90^\circ$  is called an acute angle. *Figure 1*



*Figure 1*

Each side of a right triangle has a name: *Figure 2*

1. Hypotenuse – always the longest side
2. Opposite – the side opposite angle "A"
3. Adjacent – the side next to angle "A"



*Figure 2*



The adjacent and opposite sides depend on the angle you are trying to find or use in the calculation.

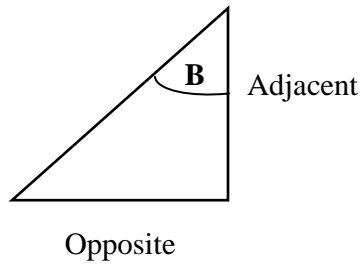


Figure 3

1. Hypotenuse – always the longest side
2. Opposite – the side opposite angle “B”
3. Adjacent – the side next to angle “B”

Right Triangle Formulas: (where “A” represents the angle of reference in degrees.

$$\sin A = \frac{\text{Opposite leg}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Adjacent leg}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Opposite leg}}{\text{Adjacent leg}}$$

**Memory aid:**

Remember the order of Sin, Cos, Tan (refer to calculator keys) then use the following mnemonic to determine the formula:

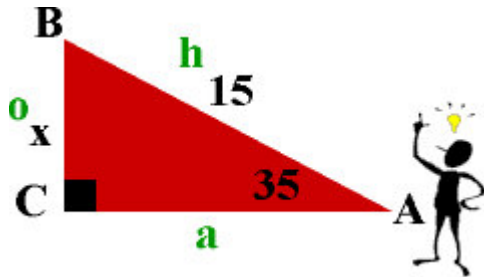
“**O**scar **H**ad **A** **H**eap **O**f **A**pples”, or

“**O**h **H**eck **A**nother **H**our **O**f **A**lgebra”



**Sin A Example Problem #1:**

In right triangle ABC, hypotenuse AB=15 and angle A=35°. Find leg BC



Sin A =  $\frac{\text{opposite leg}}{\text{hypotenuse}}$

Sin 35° =  $\frac{x}{15}$

0.5736 =  $\frac{x}{15}$

$x = 0.5736 \times 15$

$x = 8.6$

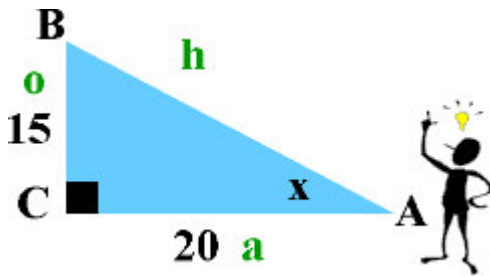
**Solution:**

1. Place the degrees in the formula for angle "A".
2. Replace "o" and "h" with their companion terms.
3. Using a scientific/graphing calculator, determine the value of the left side of the equation. *(Press 35 first and then press the sin key. If that does not work, reverse the process: sin key then 35)*
4. Solve the equation algebraically. Cross multiply and solve for "x". Or remember that if the "x" is on the top, multiply to find the answer. (Divide if "x" is on the bottom.)
5. Round answer to the required decimal place.



**Tan A Example Problem #2:**

In right triangle ABC, leg BC=15 and leg AC=20. Find angle A:



Tan A =  $\frac{\text{opposite leg}}{\text{adjacent leg}}$

$$\tan x = \frac{15}{20}$$

$$\tan x = 0.75$$

You now need to find an angle whose tangent is 0.75. To do this, use your scientific or graphing calculator. (On the scientific calculator, enter 0.75. You now need to activate the  $\tan^{-1}$  key (it is located above the  $\tan$  key). To activate this  $\tan^{-1}$  key, press **2nd** (or shift) and then the  $\tan$  key.

$$x = 36.87^\circ \text{ or } 36.9^\circ$$



**Tan B example problem #3:**

A 100-foot wharf sits along the bank of a river. A surveyor stands directly across the river from one end of the wharf. From where he stands, the angle between the lines of sight to the two ends of the wharf is  $31^\circ$ . How wide is the river?

$b$  = Width of the river

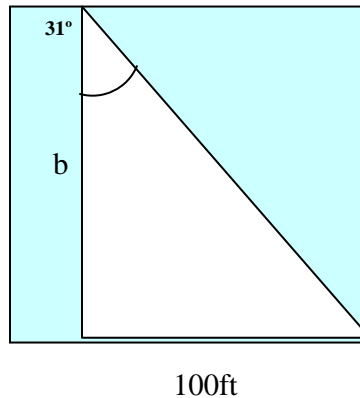
$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$

$$\tan A = \frac{100}{b}$$

$$b = \frac{100}{\tan A}$$

$$b = \frac{100}{0.60086}$$

$$b = 166.4 \text{ feet}$$



For additional reference, please visit:

[www.khanacademy.org](http://www.khanacademy.org)

- Basic trigonometry ratios
- Soh, Cah, Toa
- Using trigonometry to solve for missing information

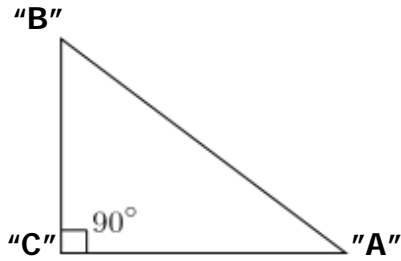
<http://msi.nga.mil>

- American Practical Navigator
  - Trigonometry, page 320
  - Trigonometric functions, page 321



***Practice Problems #8:***

Use this reference triangle to answer the following questions:



1. In right triangle ABC,  $AB = 23.8$  and angle  $A = 14.5^\circ$ . Find leg BC
2. In right triangle ABC, leg  $BC = 9.7$  and leg  $AC = 21.7$ . Find angle "A"
3. In right triangle ABC, leg  $AB = 33.8$  and Angle "B" is  $25^\circ$ . Find leg AC





**Answers to Practice Problems #8**

1. 5.96 or 6.0
2.  $24.1^\circ$
3. 14.28

